Analysing production trajectories with the exponential model

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Hubbert linearization used to estimate $U$ and $r$

$$P_c = \frac{U}{1 + \left(\frac{U}{P_{c0}} - 1\right) e^{-rt}}$$

$$\frac{P}{P_c} = -\frac{r}{U} P_c + r$$

Deviation due to mathematical artefact
Early in the cycle

\[ P_c = \frac{U}{1 + \left( \frac{U}{P_{c0}} - 1 \right)e^{-rt}} \]

\[ P_c \sim P_{c0} e^{rt} \]

\[ P \sim P_{c0} re^{rt} \]

\[ \ln P \sim rt + \ln r P_{c0} \]

\[ P \sim r P_c \]

No mathematical artefact
1) backward regressions, evolution of $r_{\text{exp}}$ in function of time

$$r_{\text{simple}} = e^{r_{\text{exp}}} - 1$$

$\sim r_{\text{exp}}$ (\(e^r = 1 + r\) pour \(r \ll 1\))

Useful representation to detect onset of trend changes
2) **central regressions, evolution of** $r_{\text{exp}}$ **in function of time**

- **Peak reached when** $r_{\text{exp}}$ **values cross the zero threshold**
3) central regressions, evolution of \( r_{\exp} \) in function of \( P_c \)

\( r_{\exp} \) values form a straight line crossing the vertical axis at \( r \) and the horizontal axis at \( U/2 \).
USA lower-48

- $r_{exp} = 8.5\%/y$
- $U_{est} = 190 \text{ Gb}$
- $P_c = 12 \text{ Gb}$

- $r = 6.5\%/y$
- $U_{est} = 190 \text{ Gb}$
- $P_c = 172 \text{ Gb}$

- $r_{exp} = 13\%/y$
- $P_c = 0.2 \text{ Gb}$
USA lower-48

High growth period

$r_{exp} > r$

post-war boom!

$r = 6.5\% / y$

XIXs crises

1930
USA lower-48

above trend until
~ 6% of $U_{est}$ (190 Gb)
has been extracted

1965

over-
under-
production

1973 1978

Trend lost,
increase of
$U_{est} > 190$ Gb

1930

$U_{est} = 190$ Gb

XIXs crises

1957

1978

r=6.5%/y

2013

Pc-1 (Gb)
above trend until
\sim 5\% of U_{est} (190 \text{ Gb})
has been extracted